DISTRIBUTED EVENT-BASED OBSERVERS FOR LTI NETWORKED SYSTEMS

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Abstract: A novel event-based technique for distributed estimation over wireless sensor networks (WNSs) is presented. The methodology is based on local Luenberger-like observers in combination with a consensus strategy. The observer design problem is firstly solved via linear matrix inequalities by assuming periodic communication among the observers. A proof of asymptotic stability of the estimation errors is provided. Then, an event–based implementation is proposed to reduce both the traffic load in the network and the energy consumption of the nodes due to unnecessary transmissions. It is shown that Globally Ultimately Uniformly Boundedness (GUUB) of the estimation errors into an arbitrary small ultimate bound region can be achieved when using the event–based implementation.

Keywords: Distributed Estimation, Event-based, Wireless Sensor Networks, Consensus, Networked Systems.

1. INTRODUCTION

Over the past decade, the development of microelectronic devices that integrate processing, sensing and communication capabilities has multiplied the number of possible control and estimation applications that can be performed in a distributed fashion. Integration of these devices allows remote monitoring and tracking in many different contexts, including large scale systems, surveillance, health care and building automation, (see, for instance Estrin et al. [1999]; Akyildiz et al. [2002]; Arranz et al. [2009]). Moreover, the technological advances in wireless communication enabled the integration of such devices allowing flexible, inexpensive, and easily configurable systems of interconnected nodes, giving birth to wireless sensor networks (WSNs).

Typical applications of WSNs consider large scale systems, where the network is generally composed by a huge number of sensors. Because of that, it appears more convenient to implement distributed estimation strategies, in which each node communicates only with a set of neighbors.

A vast literature related to the problem of distributed estimation in networked systems already exists (see, e.g. Speranzon et al. [2008]; Farina et al. [2009]; Maestre et al. [2010]). One of the most common approaches used for distributed estimation of dynamical systems exploits distributed Kalman filtering (DKF) based on consensus strategies. Such technique provides the correction of the local estimation in each node based on the information received from its neighborhood. For instance, in (Maestre et al. [2011]) the authors propose a distributed Kalman filter

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scheme where the communication strategy is devised by exploiting tools from cooperative game theory. In (Olfati-Saber and Shamma [2005]), the estimation problem is decomposed into two separate dynamic consensus problems, which are solved in a distributed fashion involving low-pass and band-pass consensus filters. In (Olfati-Saber [2007]), three novel DKF algorithms are introduced: the first is based on a consensus data fusion scheme, while the two latest are based on estimates consensus. The work in (Alriksson and Anders [2006]) also follows the strategy based on consensus on estimates.

Despite many works on distributed estimation topics, there is still room for further investigation. In particular, we believe that important and not yet satisfactorily addressed issues in the design of distributed estimation are related to the network resources. Fundamental problems such as energy efficiency or network traffic load are often neglected, and the implementation of existing estimation schemes on real networks may require high traffic rate and substantial energy demand, which is not feasible with WSNs.

On the other hand, to tackle these problems, event and self–triggered communication paradigms are being extensively used, see (Tabuada [2007]; Mazo et al. [2010]; Wang and Lemmon [2009]; Heemels et al. [2008]; Tiberi et al. [2010]; Dimarogonas and Johansson [2009]), to name a few. By using aperiodic sampling schemes, such control paradigms aim at reducing the energy consumption and the traffic load with respect to classic periodic implementations. Nevertheless, most of the work concerning event and self–triggered control are focused on control problems, while estimation is still a scarcely explored field in this context, and this paper is among the first works that address the distributed estimation problem in an event–triggered context. More in detail, here we propose a distributed event–based estimation technique that apply for Linear Time Invariant (LTI) processes in which each output is measured by a network node that exchanges information with a set of neighbors. The global observer's structure is based on local Luenberger-like observers in combination with a consensus strategy. The distributed estimation problem is firstly solved via linear matrix inequalities by assuming periodic communication. An asymptotic stability proof is derived. Then, based on the stabilizing properties of the designed observers, we address the problem of reducing both the energy expenditure and the network traffic load due to unnecessary transmissions by proposing an event-based communication policy. A proof of Globally Ultimately Uniformly Boundedness (GUUB) of the estimation error is provided.

2. NOTATION AND PRELIMINARIES

We denote with $\mathbb{R}^n$ the n-dimensional Euclidean space, with $\mathbb{R}^{n\times m}$ the set of $n\times m$ real valued matrices, with I the identity matrix, and with $\|\cdot\|$ the Euclidean vector norm or the induced matrix 2-norm as appropriate. The notation $X>0$ (respectively, $X\geq 0$), for $X\in\mathbb{R}^{n\times n}$, means that X is a positive definite (respectively, positive semi-definite) real symmetric matrix. For an arbitrarily real matrix B and two real symmetric matrices A and C, $\begin{bmatrix} A & B \\ * & C \end{bmatrix}$ denotes a real symmetric matrix, where $*$ denotes the entries implied by symmetry.

Given a system $\dot{x}=f(t,x)$, $x\in\mathbb{R}^n$, $f:\mathbb{R}^+\times D\rightarrow\mathbb{R}^n$, where f is Lipschitz with respect to x and piecewise continuous with respect to t, and where $D\subset\mathbb{R}^n$ is a domain that contains the origin, we say that the solutions are *Ultimately Uniformly Bounded* (UUB) if there exists three constants $a,b,T>0$ independent of t_0 such that for all $\|x_0\|\leq a$ it holds $\|x(t)\|\leq b$ for all $t\geq t_0+T$ and *Globally Ultimately Uniformly Bounded* (GUUB) if $\|x(t)\|\leq b$ for all $t\geq t_0+T$ and for arbitrarily large a.

The communication topology is represented with a directed graph $\mathscr{G}=(\mathscr{V},\mathscr{E})$, where $\mathscr{V}=1,2,\ldots,p$ is the set of nodes (observers) of the graph (network), and $\mathscr{E}\subset\mathscr{V}\times\mathscr{V}$ is the set of links. The set of nodes connected to node i is named the neighborhood of i, and it is denoted as $\mathscr{N}_i\triangleq\{j\in\mathscr{V}|(i,j)\in\mathscr{E}\}$.

3. PROBLEM STATEMENT

The system to observe is an autonomous linear time-invariant plant given by the following equations:

$$x(k+1)=Ax(k), \tag{1}$$

$$y_i(k)=C_i x(k), \quad \forall i\in\mathscr{V}, \tag{2}$$

where $x(k)\in\mathbb{R}^n$ is the state of the plant, $y_i(k)\in\mathbb{R}^{m_i}$ are the system outputs and p is the number of the nodes of the network.

We assume that $\forall i\in\mathscr{V}$, node i directly accesses the output y_i and that it can communicate with its neighborhood $\mathscr{N}_i$, see Fig. 1. Clearly, if the pair (A,C_i) is observable for some i, then node i is capable to reconstruct the full state x of the plant directly from y_i, without the necessity of communicate with any other node, (the *local observability* Olfati-Saber [2007]). Here, we consider instead the case of *collective observability*, that is, the systems is observable only if we put together all the nodes, i.e. the pair (A,C) is observable, where C is a matrix stacking the output matrices C_i of all the agents (Olfati-Saber [2007]).

Let's define $\bar{C_i}$ as a matrix stacking the matrix C_i and matrices C_{ij} for all $j\in\mathscr{N}_i$. It is assumed that each pair $(A,\bar{C_i})$ is observable. This is a necessary condition that impose some restrictions on the network topology and the information that is sent via each connection.

The local observer structure we use is given by the following equations:

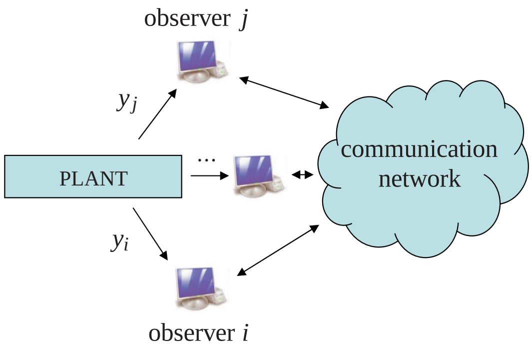


Fig. 1. System architecture.

$$\hat{x}_i(k+1) = A\hat{x}_i(k) + M_i(\hat{y}_i(k) - y_i(k)) + \sum_{j \in \mathcal{N}_i} N_{ij}(\hat{x}_j(k) - \hat{x}_i(k)), \quad (3)$$

$$\hat{y}_i(k) = C_i\hat{x}_i(k), \ \forall i = 1, 2, ..., p, \quad (4)$$

where $\hat{x}_i(k) \in \mathbb{R}^n$ is the state estimate of the observer i. With this scheme, every node tries to reconstruct the full state of the plant. Notice that each estimator is given by two pieces: a local Luenberger observer and a consensus part. The Luenberger-like observer corrects the estimated state of the plant based on the measured output $y_i(k)$ through the matrices M_i, while an additional correction is performed by consensus through the matrices N_{ij}, that take into account the information received from the neighborhood.

The problem we address in this paper is split in two parts: first, we aim at designing the matrices set $\mathscr{M} = \{M_i, i \in \mathscr{V}\}$ and $\mathscr{N} = \{N_{ij}, (i, j) \in \mathscr{E}\}$ to stabilize the observations error of every node by assuming periodic communication. Then, given the set of matrices $\mathscr{M}$ and $\mathscr{N}$ designed above, the objective is to design an event-based mechanism to reduce the amount of communication among the nodes, while ensuring GUUB of the observation error.

4. OBSERVERS DESIGN

In this section, we show how to solve the problem of distributed event–based estimation described so far. Before proceeding further, let us define the observation error of the observer i as $e_i(k) = \hat{x}_i(k) - x(k)$, i.e., the difference between the estimation of node i and the state of the plant, and let us define the vector $e(k)$ as the stack of the observation errors, i.e., $e^T(k) = [e_1^T(k) \dots e_p^T(k)]$. We remark that for periodic communication among the nodes, asymptotic stability of the closed loop will be proved, although the network is affected by waste of energy and high traffic load because of unnecessary communication. For the event–based communication, both the energy expenditure and the network traffic load are drastically reduced, but asymptotically stability is no longer achieved. However, for this second case, GUUB into an arbitrary small region is still ensured.

4.1 *Periodic case*

In the case of periodic communication, taking into account equations (1)-(4), the dynamics of $e(k)$ is given by

$$e(k+1) = (\Phi(\mathscr{M}) + \Lambda(\mathscr{N}))\, e(k), \quad (5)$$

where matrices $\Phi(\mathscr{M})$ and $\Lambda(\mathscr{N})$ depend on the sets $\mathscr{M}$ and $\mathscr{N}$, and they have the following structure:

$$\Phi = \mathrm{diag}\{A + M_1C_1, \dots, A + M_pC_p\}, \quad (6)$$

$$\Lambda(\mathscr{N}) = \sum_{(i,j) \in \mathscr{E}} \Theta_{i,j}(N_{ij}) \quad (7)$$

and

$$\Theta_{i,j}(N_{i,j}) = \begin{array}{c} \begin{array}{ccccccc} & & i & & j & & \end{array} \\ \begin{bmatrix} 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & -N_{ij} & \cdots & N_{ij} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ \\ \\ i \\ \\ \\ \end{array} \quad (8)$$

The design method resorts to a Lyapunov-based approach to prove that the procedure guarantees the asymptotic convergence of all the nodes' estimates to the actual state of the plant. Concretely, we use the following Lyapunov function:

$$V(e) = e^T P e, \quad (9)$$

where $P = \mathrm{diag}\{P_1, P_2, \dots, P_p\}$, $P_i > 0, P_i \in \mathbb{R}^n, i \in \mathscr{V}$. The following result provides a design method in terms of a Linear Matrix Inequality (LMI).

Lemma 4.1. If there exists any positive scalar $0 < \varepsilon < 1$ such that the LMI (10) has a feasible solution for positive definite matrix P, and matrices W_i, X_{ij}, $i \in \mathscr{V}$ $(i, j) \in \mathscr{E}$,

$$\begin{bmatrix} -\varepsilon P & * \\ \Phi(\mathscr{W}) + \Lambda(\mathscr{X}) & -P \end{bmatrix} < 0, \quad (10)$$

where

$$\Phi(\mathscr{W}) = P\Phi(\mathscr{M}), \mathscr{W} = \{W_i \triangleq P_i M_i, i \in \mathscr{V}\},$$
$$\Lambda(\mathscr{X}) = P\Lambda(\mathscr{N}), \mathscr{X} = \{X_{ij} \triangleq P_i N_{ij}, (i, j) \in \mathscr{E}\},$$

the estimations of all the observers asymptotically converge to the plant state by designing the observation matrices as $M_i = P_i^{-1} W_i$, $i \in \mathscr{V}$, and $N_{ij} = P_i^{-1} X_{ij}$, $i \in \mathscr{V}$, $(i, j) \in \mathscr{E}$. ◁ *Proof:* Choose the Lyapunov function (9). The forward difference can be computed as

$$\Delta V(k) = e^T(k+1) P e(k+1) - e^T(k) P e(k).$$

By taking into account equation (5) and Lemma 4.1, the forward difference can be expressed in the following way [2]:

[2] We remove the functional dependences to alleviate the notation.

$$
\begin{aligned}
\Delta V(k) &= e^T(\Phi+\Lambda)^T P(\Phi+\Lambda)e - e^T Pe \\
&\leq e^T(\Phi P+\Lambda P)^T P^{-1}(P\Phi+P\Lambda)e - e^T\varepsilon Pe.
\end{aligned}
$$

Next we impose that the Lyapunov function decreases for $e(k)\neq 0$. By using Schur complement one can obtain easily that if the next matrix inequality holds, then $\Delta V(k)<0$, which implies that the observation error of all the agents tends asymptotically to zero:

$$
\begin{bmatrix} -\varepsilon P & * \\ P\Phi(\mathscr{M})+P\Lambda(\mathscr{N}) & -P \end{bmatrix} < 0.
$$

This matrix inequality is not linear because of the products of variable matrices in the non-diagonal terms. However, from the diagonal structure of matrices P and $\Phi(\mathscr{M})$ and the special structure of matrix $\Lambda(\mathscr{N})$, given by (7) and (8), it is possible to define the change of variable in Lemma 4.1, thus obtaining the LMI (10), which concludes the proof. □

Remark 4.1. By comparing the proposed method with the works (Olfati-Saber2004,Olfati-Saber2007), our method is not restricted to use a scalar gain on the consensus part, but it introduces matrix gains which can be different for each link. ◁

4.2 *Event-based implementation*

In the previous section, we proposed a methodology to design the set of the observers gains that solve the distributed estimation problem under the assumption of periodic communication. In this section, we exploit Lemma 4.1, developing an event–based strategy to reduce both the traffic load in the network and the energy consumption of the nodes due to unnecessary transmissions. In the event–based strategy we present here, each observers decides when it is necessary to broadcast its estimates to its neighbors, based on the difference between its current estimate and the last transmitted estimate. The price to be paid in this case is that asymptotic stability is no longer guaranteed, but GUUB of the estimation errors can be proved. The trade-off between performance and communication reduction can be adjusted with an adequate tuning of a free parameter that determines the size of the ultimate bound region and the amount of communication required. The dynamics of each estimator, in the case of event–based implementation, is given by:

$$
\begin{aligned}
\hat{x}_i(k+1) &= A\hat{x}_i(k)+M_i(\hat{y}_i(k)-y_i(k)) \\
&\quad + \sum_{j\in\mathscr{N}_i} N_{ij}(\hat{x}_j(l_j)-\hat{x}_i(k)), \qquad (11)\\
\hat{y}_i(k) &= C_i\hat{x}_i(k),\ \forall i=1,2,\ldots,p, \qquad (12)
\end{aligned}
$$

where $l_j\leq k$ is the last time instant when the observer j communicated its estimated state to its neighborhood. Equation (11) takes in consideration aperiodic communication through the variable l_j, which can be distinct for each observer $i\in\mathscr{V}$. Equation (11) can be rewritten as follows

$$
\begin{aligned}
\hat{x}_i(k+1) =& A\hat{x}_i(k)+M_i(\hat{y}_i(k)-y_i(k)) \\
&+ \sum_{j\in\mathscr{N}_i} N_{ij}(\hat{x}_j(k)-\hat{x}_i(k)) + \sum_{j\in\mathscr{N}_i}\omega_j(k),
\end{aligned}
\qquad (13)
$$

where

$$
\omega_j(k) = N_{ij}(\hat{x}_j(l_j)-\hat{x}_j(k)). \qquad (14)
$$

Based on (13), the evolution of the observers with periodic communication is equivalent to the evolution with event–based communication, difference being in the terms $\omega_j(k)$, which are given by Equation (14). Every time the node j broadcasts its state to his neighborhood $\mathscr{N}_j$, we have $\omega_j(k)=0$. Therefore, $\omega_j(k)$ can be interpreted as an external perturbation due to the discontinuous flow of information between neighbors that is reset to zero at every transmission time. It is worth to point out that the disturbance that each observer j induces on its neighbors is unknown to other neighbors, but can be tracked by j, which has access to its own local estimations.

Since we are considering the designed observers matrices to satisfy the conditions of Lemma 4.1, here the matrices describing the dynamic of the error are not variables and are fixed, that is $\Phi(\mathscr{W})=\Phi$, $\Lambda(\mathscr{X})=\Lambda$. By proceeding analogously to the periodic case, the augmented observation error vector can be written as

$$
e(k+1) = \Xi e(k)+\Gamma\omega(k), \qquad (15)
$$

where

$$
\Xi=\Phi+\Lambda, \quad \omega(k)^T=[\omega_1^T\ \ldots\ \omega_p^T],
$$
$$
\Gamma(\mathscr{N}) = \sum_{(i,j)\in\mathscr{E}} \Psi_{i,j}
$$

and

$$
\Psi_{i,j} = \begin{array}{cc}
\begin{array}{ccccccc} & & & & j & & \end{array} & \\
\left[\begin{array}{ccccccc}
0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\
\vdots & & \vdots & & \vdots & & \vdots \\
0 & \cdots & 0 & \cdots & N_{ij} & \cdots & 0 \\
\vdots & & \vdots & & \vdots & & \vdots \\
0 & \cdots & 0 & \cdots & 0 & \cdots & 0
\end{array}\right] & \begin{array}{c} \\ \\ i \\ \\ \\ \end{array}
\end{array}
$$

An event–based communication policy to ensure GUUB of the error dynamics (15) is given by the following important result.

Theorem 4.1. Consider the observers (11)–(12) with gains as defined in Lemma 4.1. Suppose each node j broadcasts its estimate $\hat{x}_j$ to his neighbors $\mathscr{N}_j$ at each k such that $\|\omega_j(k)\|_\infty\geq\delta$, with ω_j defined in (14). Then, the estimation error e is GUUB with bound

$$
\|e\|_2 \leq \delta\sqrt{\frac{n\lambda_{max}(P)}{\lambda_{min}(P)}}(\alpha\|\Xi\|_\infty+\|\Gamma\|_\infty),
$$

$$\alpha = \frac{\|\Gamma^T P\Xi\|_\infty + \sqrt{\|\Gamma^T P\Xi\|_\infty^2 + (1-\varepsilon)\lambda_{\min}(P)\|\Gamma^T P\Gamma\|_\infty}}{(1-\varepsilon)\lambda_{\min}(P)}. \tag{16}$$

◁

Proof: From Theorem 4.1 and equation (15), the evolution of the Lyapunov function is given by [3]:

$$\begin{aligned}\Delta V &= [e^T\Xi^T + \omega^T\Gamma^T]P[\Xi e + \Gamma\omega] - e^T Pe \le -e^T(1-\varepsilon)Pe \\ &+ 2\omega^T\Gamma^T P\Xi e + \omega^T\Gamma^T P\Gamma\omega \le -(1-\varepsilon)\lambda_{\min}(P)\|e\|_\infty^2 \\ &+ 2\|\Gamma^T P\Xi\|_\infty\|\omega\|_\infty\|e\|_\infty + \|\Gamma^T P\Gamma\|_\infty\|\omega\|_\infty^2. \end{aligned} \tag{17}$$

The right hand side of Equation (17) is an algebraic second order equation in $\|e\|_\infty$, such that it is easy to see that the Lyapunov function $V(k)$ decreases whenever $\|e(k)\|_\infty > \alpha\|\omega(k)\|_\infty$, where α is given by equation (16). Given that each of the observers has access only to local information, and monitories the disturbance caused in its neighbors, if one takes the infinity norm of the disturbance vector ω, then it is possible to bound it bellow a desired level $\|\omega\|_\infty < \delta$ using only local information in each node. This way, it yields that $\Delta V(k) < 0$ in the region $\|e(k)\|_\infty > \alpha\delta$.

Now consider k^* as the time instant when the estimation errors enters in the region $\|e(k)\|_\infty < \alpha\delta$. Then, taking into account the error dynamics given by (15), one can easily obtain that

$$max\|e(k^*+1))\|_\infty = (\alpha\|\Xi\|_\infty + \|\Gamma\|_\infty)\delta$$

and the error can leave the region $\|e(k)\|_\infty \le \alpha\delta$. After that, the Lyapunov function needs to decrease again, so the space enclosed by maximum of the Lyapunov function in k^*+1 is an ultimate bound for the estimation error. Using the inequalities $\lambda_{min}(P)\|e\|_2^2 \le e^T Pe \le \lambda_{max}(P)\|e\|_2^2$ and $\|e\|_2 < \sqrt{n}\|e\|_\infty, \forall e \in \mathbb{R}^n$, one can easily compute the ultimate bound given in this Theorem. □

Remark 4.2. As it has been shown, the parameter δ is related with the size of the ultimate bound region of the estimation error e. By enlarging the value of δ, it is possible to reduce the amount of transmission of the nodes, while by reducing it, a better estimation performance is achieved, since the observation error is bounded in a smaller region. ◁

5. ILLUSTRATIVE EXAMPLE

In this section, we illustrate the proposed methodology by an example. Let $x := [x_1 \quad x_2 \quad x_3]^T \in \mathbb{R}^3$, and consider the system

$$x(k+1) = \begin{bmatrix} 0.95 & 0 & 0 \\ 0 & 0.809 & 1 \\ 0 & -0.3455 & 0.809 \end{bmatrix} x(k).$$

[3] We remove time indices to alleviate the notation.

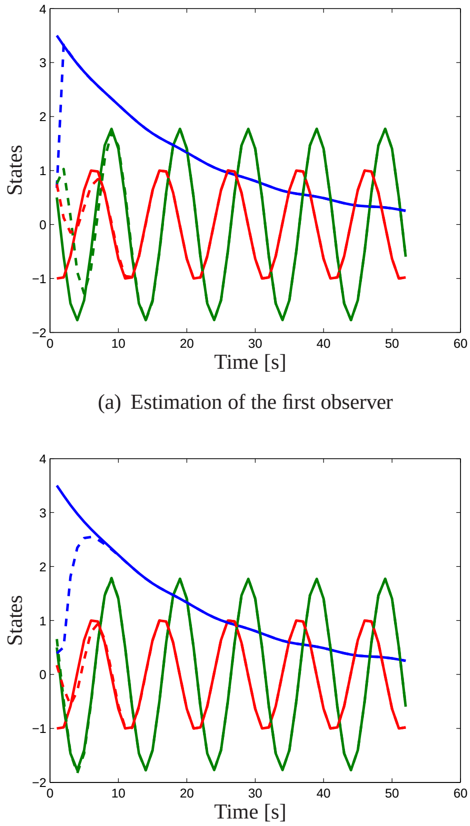
We assume that two devices are estimating the state of the plant measuring distinct outputs. Specifically, the first node has access to the first state, that is $y_1 = [1\ 0\ 0]x$, while the second node measures the output $y_2 = [0\ 1\ 1]x$.

Figure 2 illustrates the simulation results in case of periodic communications. The dotted line represents the estimations of each node, while the actual states of the plant are plotted in continuous line. It can be appreciated how the estimations of the two observers asymptotically converge to the actual states of the plant.

Figure 3 depicts instead the evolutions of the plant state and the estimations of the observers for the event-based communication policy. The triggering threshold is set to $\delta = 0.9$. It is interesting to see how in this case the estimate error of both the observers is bounded. In the periodic case we experienced 102 transmissions between the observers, while in the event–based case we counted 51 transmissions, resulting in a save of communication of 50%.

6. CONCLUSIONS

In this paper a novel distributed event–based methodology has been presented. With respect to traditional periodic implementations, event–based techniques allow a reduction of the communication among the



(a) Estimation of the first observer

(b) Estimation of the second observer

Fig. 2. Observers' estimates in the periodic case.

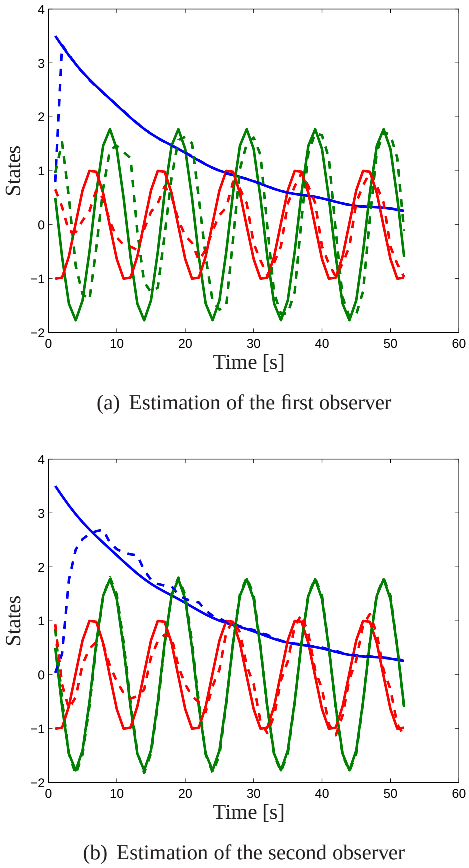


Fig. 3. Observers' estimates in the event–based case.

nodes. Such reduction of communication has the likely effect of reducing both the traffic load in the network and the energy expenditure of the nodes, still ensuring acceptable performance of the closed loop system.

Further work includes the investigation of packet dropouts and time delays in the network. Another future research direction is to consider a more general scheme with both distributed observers and distributed controllers.

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