POSICAST PID CONTROL OF OSCILLATORY SYSTEMS

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Abstract: Controlling oscillatory systems is a relevant problem due to the wide range of practical applications. This type of systems often requires small overshoot and the smallest settling response time. This design objective can be difficult to achieve, if it is conflicting with the disturbance rejection objective. A new technique is proposed here, based on a three-step Posicast input command shaping technique and proportional, integral and derivative control. The PID controller is tuned using a particle swarm optimization algorithm. Simulation experiments are presented, by using oscillatory systems with dynamics higher than second-order, with a complex pair of poles. This illustrates the potential of the proposed technique including possible extensions. © Controlo 2012

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1. INTRODUCTION

Many industrial applications can be represented by underdamped second order dynamics, such as robot control (Singhose and Seering, 2005), crane control (Sørensen et al., 2007) vibration control (Singer and Seering, 1990; Singhose, 2009; Dhanda and Franklin, 2005) and power-systems electronics (Li, 2009; Chiang et al., 2008). An important control issue is to achieve good set-point tracking for this type of systems, particularly concerning the first overshoot, which ideally should be zero. Smith (1957) proposed Posicast control as an open-loop feedforward technique to deal with underdamped systems and achieve fast responses and zero overshoot.

The Posicast principle, reviewed in section 2, is based on splitting the reference input command into several components, which are applied to the system at different time instants. The instants in which these components are applied are related to the underdamped system period of oscillation (Tallman and Smith, 1958). The Posicast control principle motivated the development of flexible systems and vibration control theory from the last 50 years (Singhose, 2009). Indeed, the half-cycle Posicast controller is quite well known in the field of flexible structures and vibration control, as the equivalent Zero Vibration (ZV) input command shaper (Singer and Seering, 1990). Some research efforts have been directed recently to Posicast Control such as the ones reported in: Sugiki and Furuta, (2006); Yildiz et al., (2010); Kalantar and Mousavi, (2010); Kucera and Hromcik, (2011).

One of the problems associated with the original Posicast control is that it was formulated for pure underdamped second-order systems. Other system dynamics, particularly with oscillatory behaviour, can also benefit significantly by using Posicast control. However, this implies to approximate critical second order parameters from the higher order process, such as the damping factor, natural frequency and
underdamped natural frequency. In this paper techniques proposed originally by Vrančić and Oliveira (2012) Oliveira and Vrančić (2012) for half-cycle Posicast are extended to design three-step Posicast controllers for oscillatory systems.

The remainder of the paper is organized as follows: section two presents an overview of Posicast Control. Section three presents the Posicast for high-order oscillatory systems. In section four, simulation results are presented. Finally, in section five some conclusions are presented and further work outlined.

2. POSICAST CONTROL: OVERVIEW

The half-cycle (hc) Posicast input command shaping structure is illustrated in Figure 1.

![Diagram of a control system](image)

Fig. 1. Half-cycle Posicast.

From this figure it is simple to derive the following transfer function, between the control signal $u$ and the input command, $r$:

$$G_{hc}(s) = A_1 + A_2 e^{-\frac{T_d}{2}} = A_1 + (1 - A_1) e^{-\frac{T_d}{2}}$$

(1)

where: $A_1$ and $A_2$ represent the amplitudes of the two unit step components and $T_d$ represent the underdamped time period. The half-cycle transfer function is often represented as follows (Hung, 2007):

$$G_{hc}(s) = \frac{1}{1 + \delta} + \frac{\delta}{1 + \delta} e^{-\frac{T_d}{2}} = 1 + P(s)$$

(2)

$$P(s) = \left(\frac{\delta}{1 + \delta}\right) \left(1 + e^{-\frac{T_d}{2}}\right)$$

(3)

with $\delta$ representing the first overshoot. The step amplitudes can be evaluated from the overshoot with:

$$A_1 = \frac{1}{1 + \delta} \quad A_2 = \frac{\delta}{1 + \delta} \quad (4)$$

A particular Posicast using three steps can be extracted from (5), which will be termed in the sequel as three-step (ts) Posicast governed by:

$$G_{ts}(s) = A_1 + A_2 e^{-T_{1s}} + A_3 e^{-T_{2s}}$$

(7)

subjected to:

$$A_1 + A_2 + A_3 = 1$$

(8)

Parameters: $A_1$, $A_2$ and $A_3$ represent the three amplitudes of the unit step components and $T_{1s}$ and $T_{2s}$ represent the step transition times. The location of the three-step Posicast zeros, which cancel the underdamped poles, depends on three amplitudes and transition times. The real and imaginary parts of:

$$A_1 e^{-T_{1s}} + A_3 e^{-T_{2s}} = 0$$

(9)

are represented respectively by:

$$A_1 + A_2 e^{-\sigma T_1} \cos(\omega T_1) + A_3 e^{-\sigma T_2} \cos(\omega T_2) = 0$$

(10)

$$A_2 e^{-\sigma T_1} \sin(\omega T_1) + A_3 e^{-\sigma T_2} \sin(\omega T_2) = 0$$

(11)

Assuming that $T_s = 2T_d$, equations (10-11) become:

$$A_1 + A_2 e^{-\sigma T_1} \cos(\omega T_1) + A_3 e^{-2\sigma T_2} \cos(\omega 2T_1) = 0$$

(12)

$$A_2 e^{-\sigma T_1} \sin(\omega T_1) + A_3 e^{-2\sigma T_2} \sin(2\omega T_1) = 0$$

(13)

From (13) it follows:

$$A_2 e^{-\sigma T_1} \sin(\omega T_1) + A_3 e^{-2\sigma T_2} \sin(2\omega T_1)$$

and

$$e^{-\sigma T_1} \sin(\omega T_1)$$

(14)

$$A_2 e^{-\sigma T_1} \sin(\omega T_1) + A_3 e^{-2\sigma T_2} \sin(2\omega T_1) = 0$$

(15)

$$A_2 = -2A_3 e^{-2\sigma T_1} \cos(\omega T_1)$$

(16)

Replacing (16) in (12) leads to the following expressions:

$$A_1 - 2A_3 e^{-2\sigma T_1} \cos^2(\omega T_1) + A_3 e^{-2\sigma T_2} \cos(2\omega T_1) = 0$$

(17)

$$A_1 - 2A_3 e^{-2\sigma T_1} \cos^2(\omega T_1) + A_3 e^{-2\sigma T_2} [\cos^2(\omega T_1) - \sin^2(\omega T_1)] = 0$$

(18)

$$A_1 - A_3 e^{-2\sigma T_1} [\cos^2(\omega T_1) + \sin^2(\omega T_1)] = 0$$

(19)

$$A_1 - A_3 e^{-2\sigma T_1} = 0$$

(20)

Replacing (20) and (16) in (12) gives the expression for $A_2$:

$$A_3 e^{-2\sigma T_1} - 2A_3 e^{-\sigma T_1} \cos(\omega T_1) + A_3 = 1$$

(21)

$$A_3 = \frac{1}{1 - 2e^{-\sigma T_1} \cos(\omega T_1) + e^{-2\sigma T_1}}$$

(22)

Parameter $A_2$ is evaluated by inserting (22) into (16):

$$A_2 = \frac{-2e^{-\sigma T_1} \cos(\omega T_1)}{1 - 2e^{-\sigma T_1} \cos(\omega T_1) + e^{-2\sigma T_1}}$$

(23)

Replacing (22) in (19):

$$A_1 = \frac{e^{-2\sigma T_1}}{1 - 2e^{-\sigma T_1} \cos(\omega T_1) + e^{-2\sigma T_1}}$$

(24)
Considering that:
\[ \sigma = -\zeta \omega_n, \quad \omega = \omega_d \]  \hspace{1cm} (25)
and replacing (25) in (24), (23) and (22), result in:
\[ A_1 = \frac{e^{2\zeta \omega_n T_1}}{1 - 2e^{\zeta \omega_n T_1} \cos(\omega_d T_1) + e^{2\zeta \omega_n T_1}} \]  \hspace{1cm} (26)
\[ A_2 = \frac{-2e^{\zeta \omega_n T_1} \cos(\omega_d T_1)}{1 - 2e^{\zeta \omega_n T_1} \cos(\omega_d T_1) + e^{2\zeta \omega_n T_1}} \]  \hspace{1cm} (27)
\[ A_3 = \frac{1}{1 - 2e^{\zeta \omega_n T_1} \cos(\omega_d T_1) + e^{2\zeta \omega_n T_1}} \]  \hspace{1cm} (28)

If \( T_1 = T_2/6 \) then \( T_2 = T_1/3 \), this corresponds to the one-third-cycle case; if \( T_1 = T_2 / 8 \) then \( T_2 = T_1 / 4 \), this corresponds to the one-quarter-cycle case, If \( T_1 = T_2 / 10 \) then \( T_2 = T_1 / 5 \), this corresponds to the one-fifth-cycle case, and so on.

3. POSICAST FOR HIGHER-ORDER OSCILLATORY SYSTEMS

One of the problems of Posicast input command shaping (PICS), is that its performance highly depends on a good estimation of the second order underdamped model parameters. This is even more pronounced when the dynamics to be controlled are represented by models with order higher than second-order. Indeed, if one tries to approximate higher order oscillatory systems dynamics, by using the second order model, this may not result well within the PICS framework. In the case of high-order systems with one pair of complex poles, the PICS can be designed assuming the complex dominant poles. Alternatively, a new heuristic method to extract the half-cycle Posicast key design parameters (overshoot, \( \delta \), and the underdamped time constant \( T_d \), was proposed by Vrančić and Oliveira, (2012), for systems with one pair of complex poles. The proposed technique consists of measuring three successive peaks from the process open-loop response as illustrated in Figure 2:

The Posicast parameters are evaluated from the amplitudes and times extracted from the open-loop response using the following expressions:
\[ A_1 = d_1 / (d_1 + d_2) \quad T_d \approx t_2 - t_1 \]  \hspace{1cm} (29)

From these two parameters it is possible to obtain the overshoot using (4):
\[ \delta = \frac{1}{A_1} - 1 \]  \hspace{1cm} (30)
and:
\[ \zeta = \frac{\ln^2(\delta)}{\pi^2 + \ln^2(\delta)} \]  \hspace{1cm} (31)
\[ \omega_d = \frac{2\pi}{T_d} \]  \hspace{1cm} (32)
\[ \omega_n = \frac{2\pi}{T_d \sqrt{1 - \zeta^2}} \]  \hspace{1cm} (33)

To illustrate the proposed PICS design methodology consider the following process transfer function:
\[ G_{p1}(s) = \frac{e^{-s}}{(1 + 0.5s + 2s^2)(1 + s)} \]  \hspace{1cm} (34)

From the open-loop response by using expressions (29-33) the following parameters are obtained:
\[ A_1 = 0.637, \quad T_d = 9.02s \]  \hspace{1cm} (35)
\[ \delta = 0.57, \quad \zeta = 0.176 \]  \hspace{1cm} (36)
\[ \omega_n = 0.707\text{rad}/s, \quad \omega_d = 0.697\text{rad}/s, \]  \hspace{1cm} (37)
resulting in the following half-cycle transfer function:
\[ G_{hc1}(s) = 0.637 + 0.363e^{-4.51s}, \]  \hspace{1cm} (38)
third-cycle (hc), quarter-cycle (qc) and fifth-cycle (fc) transfer functions:
\[ G_{hc1}(s) = 1.165 - 0.966e^{-1.503s} + 0.801e^{-3.007s} \]  \hspace{1cm} (39)
\[ G_{qc1}(s) = 1.900 - 2.335e^{-1.128s} + 1.435e^{-2.255s} \]  \hspace{1cm} (40)
\[ G_{fc1}(s) = 2.836 - 4.100e^{-0.902s} + 2.264e^{-1.804s} \]  \hspace{1cm} (41)

![Fig. 2. Open loop process response for an higher-order process with one pair of complex poles.](image)

![Fig. 3. Open-loop response for system \( G_{p1} \), with half-cycle, third-cycle, quarter-cycle and fifth cycle Posicast (38-41).](image)
Fig. 4. Open-loop response shaped signals for $Gp_I$, with half-cycle, third-cycle, quarter-cycle and fifth cycle Posicast (38-41).

Figures 3 and 4 present the open-loop response and shaped signals for process $Gp_I$, achieved with PICS transfer functions (38-41).

4. POSICAST WITH FEEDBACK CONTROL

From the results presented in section 3 it is clear that PICS can be used to achieve deadbeat set-point tracking responses. Posicast was originally proposed to be incorporated into a Feedback loop by Smith (1957), as depicted in Figure 5, in which the transfer function $P$ is governed by (3).

$$G_p(s) = \frac{s^2K_d + sK_p + K_i}{s(1 + sT_f)}$$

Fig. 5. Posicast used as a pre-filter in a two-degree of freedom configuration.

The classical ideal design of this type of two-degrees of freedom control loop by using a pre-filter as a lead-lag compensator requires that both the pre-filter and controller to be designed at the same time. However, it is more usual (Skogestad and Postlethwaite, 1996), to design first the feedback controller to achieve a good disturbance rejection and then the pre-filter for enhancing set-point tracking. In the case of PICS, the ideal design can be used for the shaper, but it may not be the most suitable in some cases.

More recently, Posicast has been used within the Feedback loop (Hung, 2007; Huey et al., 2008; Kucera and Hromcik, 2011) as presented in Figure 6.

Fig. 6. Posicast used within the feedback loop.

Recently a dual mode control configuration was proposed by Oliveira and Vrancic, (2012) illustrated in Figure 7, using half-cycle Posicast.

This paper proposes using three-step Posicast (7) as the feedforward controller. The general principle of this control structure is to use the PICS for set-point tracking with pure feedforward control and switch to automatic when the transient response has settled. The regulation is then performed by the feedback loop, in this case using as PID controller. The great disadvantage of this configuration is the inexistence of regulation action in the tracking phase.

The control structures presented in Figures 5, 6 and 7 are labeled hereby: I, II and III, respectively, and will be compared by means of simulation in the following section.

4. ILLUSTRATIVE EXAMPLE

In the examples presented in this section the PICS is designed using the technique described in section 3 and the PID controller is tuned by using a particle swarm optimization algorithm (PSO) (Oliveira et al, 2011). The heuristics used in the PSO algorithm were the following: swarm size of 30 particles, the cognitive and social constants set to 2, and inertia weight linearly decreasing between 0.9 and 0.7 over 100 iterations. The following simulations consider the PSO design of a PID controller for disturbance rejection, with the following structure:

$$G_c(s) = \frac{s^2K_d + sK_p + K_i}{s(1 + sT_f)}$$

where: $K_p$, $K_i$ and $K_d$ represent the proportional, integral and derivative term gains, respectively, and $T_f$ the filter time constant. In all the cases presented here $T_f=0.1$. In configurations I and III, the following PID gains were used, obtained with a PSO algorithm:

$$K_p = 0.01, K_i = 0.197, K_d = 0.652$$

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Figure 8 presents the response to a unit step applied to the reference input and a step of 0.2 applied to the load disturbance input, considering configuration I. The results presented in Figure 8, show that PID controller designed for disturbance rejection, and modifies the PICS set-point tracking trajectory. Using configuration II, the following PID gains were obtained with the PSO algorithm, taking into account the optimization objective of set-point tracking: \[ K_p = 0.728, \quad K_i = 0.289, \quad K_d = 0.694 \] \[ (44) \]

Configuration III is shown in Figure 9, as proposed in (Oliveira and Vrancic, 2012) with an anti-windup and bumpless transfer protection using the conditioning technique (Hanus et al., 1987; Walgama et al., 1992).

In Figures 10 and 11 the results obtained with configuration II and III were superimposed for easier comparison. These simulations show the system (34) response to a unit step applied to the reference input (at \( t=1s \)) and a step of 0.2 applied to the load disturbance input (at \( t=35s \)). As it can be seen for case II, set-point tracking improved and the disturbance rejection performance got significantly worst, when compared to the one in Figure 8. The results obtained with configuration presented in Figure 9, for system (34), show that the system tracks the set-point with a deadbeat-like response switching (at \( t_s=10s \)) to the PID feedback controller which rejects the load disturbance. In this case the transition is bumpless between manual and automatic.

The results presented in Figure 11 between cases II and III, show similar performance in terms of set-point tracking. However, in terms of disturbance rejection, the switching configuration III it is clearly superior, as the PID controller used was designed for this objective.

The advantage of the switching-based feedforward-feedback configuration is that it can use the three-step Posicast shapers represented by (7). Therefore it improves the set-point tracking performance and after switching to automatic mode it has good disturbance rejection.

The incorporation of the three-step shaper within the feedback loop using the configuration II may be problematic in two aspects: i) the control signal magnitude increase, due to the simultaneous action of the PICS and PID; The three step PICS, depending on the version used, can be much more aggressive in terms of actuator effort that the half-cycle PICS : ii) if the PID controller is designed for set-point tracking, depending on the systems dynamics, it may perform not optimally for disturbance rejection, and vice-versa.

Figure 12 presents the results of using the third-cycle PICS represented by (39).

Figure 12 presents the results of using the third-cycle PICS represented by (39) in configuration II with PID gains designed for set-point tracking:

\[ K_p = 0.740, \quad K_i = 0.350, \quad K_d = 0.570 \] \[ (45) \]
As it can be seen the performance for disturbance rejection is not satisfactory.

5. CONCLUSIONS AND FURTHER WORK

Three-step based Posicast input command shaping and PID control were proposed within a switching technique to deal both with set-point tracking and disturbance rejection. The plant dynamics considered are oscillatory, with one pair of complex poles and order higher than two. For this type of oscillatory systems, the proposed technique allows a simple and effective way of estimating the key Posicast design parameters from the open-loop step response: the underdamped period of oscillation and the first overshoot. This allows designing third-cycle, quarter-cycle, fifth-cycle and any other fraction of the underdamped time period, as long as the actuator can cope with the physical control effort demands. The simulation results presented shows the effectiveness of the proposed technique.

More research effort has to be devoted to develop efficient ways to incorporate the three steps Posicast within the feedback loop, taking into account robustness issues. Higher order systems with more than one pair of complex poles require the use of more complex Posicast shapers (e.g., staggered Posicast). The former type of oscillatory systems requires the extension of the proposed technique.

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